SYLLABUS: INTRODUCTION TO ABSTRACT ALGEBRA

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Course Description

Does high school math seem too simple? Bored with trigonometry and logarithms? Then this is the course for you. Starting with some fairly simple notions like operations and properties of sets, we'll explore the beautiful area of math called abstract algebra. In particular, the course begins with groups and their properties, and then moves to rings and fields, finishing up with two big bangs: the proof that you can't solve a general fifth-degree polynomial by radicals, and a proof that it's possible to move a sphere around using only rotations to get two copies of the same sphere you started with.

Background: This will be a pretty hard course. Fluency with high school algebra and familiarity with functions, trig, and some matrices is strongly suggested.

Homework: Will probably be assigned weekly. Optional, of course, but I'll read/correct solutions if you submit them and you'll certainly get a better handle on the material if you at least work on the problems, even without solving them. If you have comments about the assignments that would make you more likely to do them (i.e., "these problems are impossible, give me something I can actually do") I'll be glad to change it up.

Course Outline

- Week 1: Notation and intro to groups
  - Notation and quick definitions: sets, union, intersection
  - Symmetries of a square
  - Isomorphisms with matrix multiplication
  - (Literally) infinite examples
- Week 2: Subgroups, homomorphisms
  - Subgroups: definition and examples
  - Other subgroups: trivials, intersection of subgroups
  - Homomorphisms
  - Kernels and the First Isomorphism Theorem
- Week 3: Lagrange's theorem, generators and relations
  - Cosets: Move around the subgroups (flip C_n)
  - Lagrange's Theorem and proof
  - Look back at C_n: generated by r with relation $r^n = 1$; D_n gen. by r, f with $r^n = 1, frfr = 1$
  - In general: cross out all the relations; use $F_1, F_2, \langle a, b\rangle/(aba^{-1}b^{-1})$; Cayley
- Week 4: Group actions and Orbit-Stabilizer
  - Group actions: definition, examples
  - Orbit-Stabilizer Theorem
  - Platonic solids and their symmetry groups, part I
- Week 5: Non-Euclidean geometry
  - Spherical and hyperbolic geometry
  - Triangles in non-Euclidean spaces
  - Triangle tilings, duals, and Cayley graphs

Date: Summer 2008.

- Week 6: Conjugacy classes
  Normal subgroups
  Conjugates and geometry
  Platonic solids, part II
- Week 7: Miscellany (some subset of the below)
  Cauchy and Sylow theorems
  Complex numbers and quaternions
- Week 8: Banach-Tarski, and some philosophy
  Review: Cayley graphs, group action, F_2
  Paradoxical decompositions, of F_2 and otherwise
  Countability, Axiom of Choice
  The Banach-Tarski Paradox
  Philosophical meanderings on the Axiom of Choice
  Wrap-up