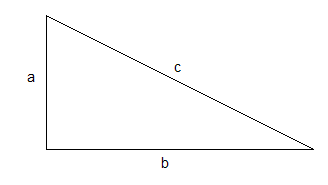
Topics in Mathematics

Pythagorean Theorem Lecture Summary (Written and Presented by Andrew Spieker)

Hopefully you all know the famous Pythagorean Theorem, which relates the sides of a right triangle in the following way:

If I label the “legs” of the right triangle as “a” and “b,” and the “hypotenuse” as “c,” then:

A nice little fact: of all the theorems in the world, this is the one to which we have the most proofs. This particular webpage gives about 80 different proofs of it:

<http://www.cut-the-knot.org/pythagoras/index.shtml>

**Then, we completed the Pythagorean Theorem Proof Exercise; the answers are posted in our documents.**

We proved that if is a primitive Pythagorean Triple (PPT,) then so is . Here is the proof:

Suppose is a PPT. Then

Multiply the equation through by and get that

We rewrite that as

Therefore is a Pythagorean Triple.

Then, we came up with a *generating* function for Pythagorean Triples:

We note that if we have a PPT, , that , or

We factor the right hand side, and get that . Since is a PPT, that means that have NO common divisors, or they are *coprime*. Therefore, and have no common divisors. BY THE WAY: I didn’t prove this in class, because my mind was drawing a blank; here is the proof:

Proof: Suppose and have common factors. This means that for some integer , divides and divides . If this is the case, then divides (by adding the equations), and divides (by subtracting the equations). Then, this implies that and have a common factor, which we sais wasn’t true. So, therefore, our original assumption that and have common factors was not true. Therefore, and have NO common factors!

The only way that two numbers without common divisors could ever be a perfect square is if they were both perfect squares to begin with. Why is this true? See if you can prove this one on your own ☺

Then, we rewrite as and as with coprime and . We solve this equation for and and we get the following function that generates EVERY SINGLE Primative Pythagorean Triple: