MIT ESP Delve
AP Calculus AB
September 17, 2011

Problem Set 1

1. Find the limit numerically i.e. construct a table, then confirm your answer with a graph. State whether the limits exists.

   (a)
   $$\lim_{x\to 3} \frac{\frac{1}{x+1} - \frac{1}{4}}{x-3}$$

   (b)
   $$\lim_{\alpha\to 4} \frac{\frac{1}{\alpha+1} - \frac{4}{5}}{\alpha-4}$$

   (c)
   $$\lim_{\phi\to 0} \frac{\sin\phi}{\phi}$$

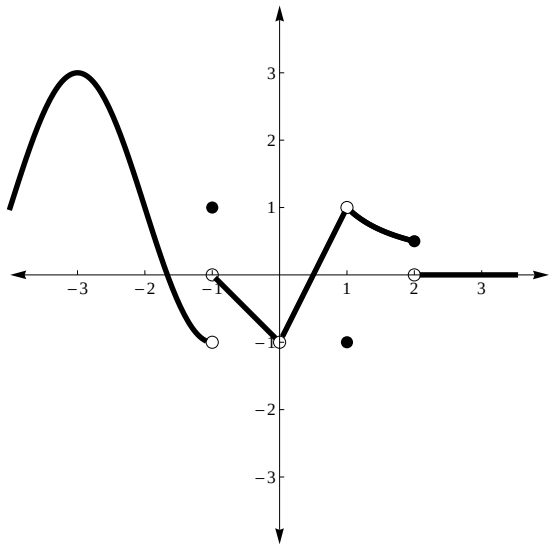
   Note: Use radians, not degrees.

   (d)
   $$\lim_{a\to 0} \frac{\cos a - 1}{a}$$

   (e)
   $$\lim_{x\to \pi/2} \tan x$$

   (f)
   $$\lim_{x\to 5} \frac{|x-5|}{x-5}$$

2. Consider the following graph of $f(x)$:



   Find the limits of $f(x)$ and the value of $f(x)$ at $x = -3, 1, 0, 1, 2$. State whether the limits exist.

3.
$$f(x) = \begin{cases} x^2, & x \le 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \ge 4 \end{cases}$$

   Find the limit as x approaches 2 and the limit as x approaches 4.

4. Identify three types of behavior associated with the nonexistence of a limit. Illustrate each type with a graph of a function.

5. (a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2?

   (b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$?

6. Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

   (a) If f is undefined at $x = c$, then the limit of $f(x)$ as x approaches c does not exist.

   (b) If the limit of $f(x)$ as x approaches c is 0, then there must exist a number k such that $f(k) < 0.001$.

7. Consider the function $f(x) = (1 + x)^{1/x}$. Estimate
$$\lim_{x\to 0} (1 + x)^{1/x}$$
   by evaluating x-values near 0. Sketch the graph of f. *Hint: You should have some experience with this number. It is transcendental.*