Breaking Rules in Math

Welcome!

A 270° Triangle

- What do a triangle's angles usually add up to?

A 270° Triangle

- What do a triangle’s angles usually add up to? 180°

A 270° Triangle

- See Google maps

Does 11/2=12?

- Definition of division: $a/b = q$ when $a = b \cdot q$
- Goal: show that $11 = 2 \cdot 12$

Does 11/2=12: Modular Arithmetic

- Think about how the teacher counts when picking groups:

  1, 2, 3, 1, 2, 3, 1, 2, 3, ...
- We pick a limit (the modulus) and start over at the limit

Does 11/2=12: Modular Arithmetic

- Think about how the teacher counts when picking groups:

  1, 2, 3, 1, 2, 3, 1, 2, 3, ...
- We pick a limit (the modulus) and start over at the limit

Mod 1:	0	0	0	0	0	0	...
Mod 2:	0	1	0	1	0	1	...
Mod 3:	0	1	2	0	1	2	...
Mod 4:	0	1	2	3	0	1	...
...							

Does 11/2=12: Modular Arithmetic

- Addition is repeated counting

- Examples with modulus 13:
- (1 + 2) mod 13 = 3
- (12 + 1) mod 13 = 0
- (11 + 5) mod 13 = 3

Does 11/2=12: Modular Arithmetic

- Addition is repeated counting

- Examples with modulus 13:
- $(1 + 2) \bmod 13 = 3$
- $(12 + 1) \bmod 13 = 0$
- $(11 + 5) \bmod 13 = 3$
- $(1 + 2) = 0 \cdot 13 + 3$
- $(12 + 1) = 1 \cdot 13 + 0$
- $(11 + 5) = 1 \cdot 13 + 3$

Does 11/2=12: Modular Arithmetic

- Multiplcation works the same way

- Examples with modulus 13:
- $(3 \cdot 2) \bmod 13 = 6$
- $(12 \cdot 3) \bmod 13 = 10$
- $(3 \cdot 2) = 0 \cdot 13 + 6$
- $(12 \cdot 3) = 2 \cdot 13 + 10$

Does 11/2=12: Modular Arithmetic

- A few more Examples with modulus 13:
- $7 \cdot 8 = ?$
- $9 \cdot 4 = ?$
- $6 \cdot (5 + 8) = ?$
- $4 + (11 \cdot 12) = ?$

Does 11/2=12: Modular Arithmetic

- A few more Examples with modulus 13:
- $7 \cdot 8 = 56 = 4 \cdot 13 + 4 = 4$
- $9 \cdot 4 = ?$
- $6 \cdot (5 + 8) = ?$
- $4 + (11 \cdot 12) = ?$

Does 11/2=12: Modular Arithmetic

- A few more Examples with modulus 13:
- $7 \cdot 8 = 56 = 4 \cdot 13 + 4 = 4$
- $9 \cdot 4 = 36 = 2 \cdot 13 + 10 = 10$
- $6 \cdot (5 + 8) = ?$
- $4 + (11 \cdot 12) = ?$

Does 11/2=12: Modular Arithmetic

- A few more Examples with modulus 13:
- $7 \cdot 8 = 56 = 4 \cdot 13 + 4 = 4$
- $9 \cdot 4 = 36 = 2 \cdot 13 + 10 = 10$
- $6 \cdot (5 + 8) = 6 \cdot 0 = 0$
- $4 + (11 \cdot 12) = ?$

Does 11/2=12: Modular Arithmetic

- A few more Examples with modulus 13:
- $7 \cdot 8 = 56 = 4 \cdot 13 + 4 = 4$
- $9 \cdot 4 = 36 = 2 \cdot 13 + 10 = 10$
- $6 \cdot (5 + 8) = 6 \cdot 0 = 0$
- $4 + (11 \cdot 12) = 4 + 2 = 6$

Does 11/2=12? Recap

- Definition of division: $a/b = q$ when $b = a \cdot q$
- Goal: show that $11 = 2 \cdot 12 \pmod{13}$

$$2 \cdot 12 = 24 = 1 \cdot 13 + 11 = 11$$

How many numbers are there?

- We have to get more specific (what kind of number)
- Let’s start with the natural numbers: 0, 1, 2, 3, ….

How many numbers are there?

- We have to get more specific (what kind of number)
- Let’s start with the natural numbers: 0, 1, 2, 3, ….
- Not a very good answer to this question!

How many numbers are there?

- What about integers? (…, -2, -1, 0, 1, 2, …)
- No absolute answer, but we can compare them to naturals

Question: Are there more integers than naturals, fewer integers, or the same amount of each?

How many numbers are there?

- We compare the sizes of the two collections by trying to match them together

First try:

Integers	...	-2	-1	0	1	2	...
Naturals	?	?	?	0	1	2	...

How many numbers are there?

- We compare the sizes of the two collections by trying to match them together

Second try:

Integers	...	-2	-1	0	1	2	...
Naturals	...	3	1	0	2	4	...

How many numbers are there?

- We compare the sizes of the two collections by trying to match them together

Second try:

Integers	0	-1	1	-2	2	...
Naturals	0	1	2	3	4	...

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- There are too many! But how can we prove it?

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- There are too many! But how can we prove it?

We need to show that however you match them up, you missed at least one decimal number

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- Goal: find the missing number in the table

N_0 =	8	6	.4	3	2	...
N_1 =	0	2	.7	7	7	...
N_2 =	8	0	.0	0	0	...
N_3 =	0	0	.0	8	2	...
...						

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- Goal: find the missing number in the table

Missing: 9

N_0 =	8	6	.4	3	2	...
N_1 =	0	2	.7	7	7	...
N_2 =	8	0	.0	0	0	...
N_3 =	0	0	.0	8	2	...
...						

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- Goal: find the missing number in the table

Missing: 95

N_0 =	8	6	.4	3	2	...
N_1 =	0	2	.7	7	7	...
N_2 =	8	0	.0	0	0	...
N_3 =	0	0	.0	8	2	...
...						

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- Goal: find the missing number in the table

Missing: 95.6

N_0 =	8	6	.4	3	2	...
N_1 =	0	2	.7	7	7	...
N_2 =	8	0	.0	0	0	...
N_3 =	0	0	.0	8	2	...
...						

How many numbers are there?

- What about decimals? (e.g. 0.6, 4, $3.\overline{33}$, 3.141592...)
- Goal: find the missing number in the table

Missing: 95.66...

N_0 =	8	6	.4	3	2	...
N_1 =	0	2	.7	7	7	...
N_2 =	8	0	.0	0	0	...
N_3 =	0	0	.0	8	2	...
...						

How many numbers are there?

- Bonus: what about fractions? (e.g. 1/3, -5/12, 10/4)
- More than naturals? Less than decimals?

How many numbers are there?

- Bonus: what about fractions? (e.g. 1/3, -5/12, 10/4)
- Same number of fractions as naturals!
- Decimals (real numbers) can go on forever, which is why there are more

Class problems

Bonus: Commutativity

Define a +’ b = a – b

Example: (4 +’ 5) = -1

but (5 +’ 4) = 1

Bonus: Associativity

Define $a +' b = 2 \cdot a + b$

Example: $(4 +' 2) +' 3 = 10 +' 3 = 23$

but $4 +' (2 +' 3) = 4 +' 7 = 15$.