Exercises and Solutionso – Become a LaTeXer!

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Splash 2009

If you have trouble figuring out how to typeset one of the following, try to figure it out using the Internet before asking us how to typeset it. Knowing how to find the information you want on the Internet is an important skill. If you're stuck or frustrated, feel free to ask for help.

Note that `http://detexify.kirelabs.org/classify.html` is useful.

1 Easy

$$e^{i\pi} + 1 = 0 \tag{1}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{2}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{3}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

$$\vec{L} = \vec{r} \times \vec{p} \tag{5}$$

$$\sqrt[3]{2} \tag{6}$$

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r} \tag{7}$$

$$\sqrt{\frac{a_1^2 + \cdots + a_n^2}{n}} \geq \frac{a_1 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 \cdots a_n} \geq \frac{n}{\frac{1}{a_1} + \cdots + \frac{1}{a_n}} \tag{8}$$

$$|\langle x, y\rangle|^2 \leq \langle x, x\rangle \cdot \langle y, y\rangle \tag{9}$$

$$\begin{aligned} &\text{A1: } \varphi \longrightarrow (\psi \to \varphi) \\ &\text{A2: } (\varphi \to (\psi \to \theta)) \longrightarrow ((\varphi \to \psi) \to (\varphi \to \theta)) \\ &\text{A3: } (\neg\varphi \to \neg\psi) \longrightarrow (\psi \to \varphi) \end{aligned} \tag{10}$$

2 Medium

$$1_A = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \tag{11}$$

$$n\underbrace{\uparrow \cdots \uparrow}_{n} n = n \to n \to n \tag{12}$$

$$\begin{aligned} 1 \uparrow 1 &= ^1 1 = 1 \\ 2 \uparrow\uparrow 2 &= ^2 2 = 4 \end{aligned}$$

$$3 \uparrow\uparrow\uparrow 3 = {}^{^3}3 = 3 \uparrow\uparrow 3 \uparrow\uparrow 3 = \underbrace{3^{3^{3^{3^{3^{\cdot^{\cdot^{\cdot^{3}}}}}}}}}_{3^{3^3}\text{ threes}} \tag{13}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{14}$$

$$\mathrm{H_2O}(\ell) + \mathrm{H_2O}(\ell) \rightleftharpoons \mathrm{H_3O^+}(aq) + \mathrm{OH^-}(aq) \tag{15}$$

$$\Gamma(n+1) \stackrel{\text{def}}{=} \int_0^\infty e^{-t} t^n \mathrm{d}t \tag{16}$$

$$\gcd(n, m \bmod n); \quad x \equiv y \pmod{b}; \quad x \equiv y \mod c; \quad x \equiv y \pmod{d} \tag{17}$$

In the following, note the bold symbols.

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \tag{18}$$

For the following exercise, you will need to `\usepackage{esint}` to get the symbol ∯.

$$\begin{aligned}
\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} &= \frac{Q(V)}{\varepsilon_0} \\
\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} &= 0 \\
\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} &= -\frac{\partial \Phi_{B,S}}{\partial t} \\
\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} &= \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}
\end{aligned} \tag{19}$$

You might find the environments `bmatrix` and `pmatrix` useful for the following exercises.

$$\rho_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{20}$$

$$\left[\begin{array}{c|ccc} 1 & 0 & \cdots & 0 \\ \hline 0 & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \cdots & * \end{array}\right] = \boxed{\begin{array}{c|ccc} 1 & 0 & \cdots & 0 \\ \hline 0 & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & * & \cdots & * \end{array}} \tag{21}$$

Note the locations of the bounds on the summation in the following exercise.

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N} p_i (x_i - \bar{x})^2} = \sqrt{\frac{\sum_{i=1}^{N} p_i (x_i - \bar{x})^2}{N}} \tag{22}$$

$$\varphi(n) = n \cdot \prod_{\substack{p \mid n \\ p \text{ prime}}} \left(1 - \frac{1}{p}\right) \tag{23}$$

If you `\usepackage{mathtools}`, you can make it look like

$$\varphi(n) = n \cdot \prod_{\substack{p \mid n \\ p \text{ prime}}} \left(1 - \frac{1}{p}\right) \tag{24}$$

Parts of the following exercise require you to `\usepackage{mathtools}`.

$$\begin{array}{ccccc} {}^{4}_{12}\mathrm{C}^{5+}_{2} & {}^{14}_{2}\mathrm{C}^{5+}_{2} & {}^{4}_{12}\mathrm{C}^{5+}_{2} & {}^{14}\mathrm{C}^{5+}_{2} & {}_{2}\mathrm{C}^{5+}_{2} \end{array} \tag{25}$$

In the following, note the size of the /, and the spacing on the sides of the |.

$$\mathbb{Q} \cong \left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z} \text{ and } b \neq 0 \right\} \Big/ \sim$$

$$\frac{a}{b} \sim \frac{c}{d} \iff ad - bc = 0 \tag{26}$$

Notice both the horizontal and vertical spacing in the following exercise.

$$\begin{aligned} 1 \uparrow 1 &= {}^{1}1 = 1 \\ 2 \uparrow\uparrow 2 &= {}^{2}2 = 4 \\ 3 \uparrow\uparrow\uparrow 3 &= {}^{3}3 = 3 \uparrow\uparrow 3 \uparrow\uparrow 3 = \underbrace{3^{3^{3^{3^{3^{\cdot^{\cdot^{\cdot^{3}}}}}}}}}_{3^{3^{3}} \text{ threes}} \end{aligned} \tag{27}$$

3 Hard

The command `\newcommand{\`*name*`}[`*n*`][`*default*`]{`*definition*`}` defines a new command, where n is the number of parameters and *default* is the default value for the first parameter. Parameters in braces ({ }) are required, and parameters in square brackets ([ ]) are optional. The parameters can be referred to via `#1`, `#2`, ..., `#9`. Using `\newcommand{\mathset}[2][`*default value for the first argument*`]{`*command definition*`}`, define a command `\mathset` that acts as follows. Notice, in particular, the sizes of the middle bar, the sizes of the braces, and the spacing between the middle bar and the things on each side.

`\mathset{1}` gives $\{1\}$

`\mathset[x]{0\leq x\leq 1}` gives $\{x \mid 0 \leq x \leq 1\}$

`\mathset[(x)_i]{\sum_i x_i \in A}` gives $\left\{ (x)_i \,\middle|\, \sum_i x_i \in A \right\}$

`\mathset[\sum_{i=1}^{\infty} n^{-s}]{n \in A}` gives $\left\{ \sum_{i=1}^{\infty} n^{-s} \,\middle|\, n \in A \right\}$

`\mathset{}` gives $\{\}$

`\mathset[\frac{1}{1+\frac{1}{x}}]{x\in A}` gives $\left\{ \frac{1}{1+\frac{1}{x}} \,\middle|\, x \in A \right\}$

`\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{x}}}]{x\in A}` gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{x}}} \,\middle|\, x \in A \right\}$

`\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}]{x\in A}` gives $\left\{ 1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}} \,\middle|\, x \in A \right\}$

`\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}]{x\in A}` gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}} \,\middle|\, x \in A \right\}$

`\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}]{x\in A}` gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}} \,\middle|\, x \in A \right\}$

`\mathset[\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}}]{x\in A}` gives $\left\{ \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}}}} \,\middle|\, x \in A \right\}$

4 Insane

Write a command `\outputcode` which outputs the code of the document being typeset.

The following exercise idea is taken from the TEXbook. Define a command `\primes{`n`}` which typesets the first n primes. For example, `\primes{30}` should give 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, and 113.

5 Diabolical

The Ackermann function is defined as

$$A(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ A(m-1,1) & \text{if } m > 0 \text{ and } n = 0 \\ A(m-1, A(m, n-1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases}$$

Define a command, `\ackermann{`m`}{`n`}`, which computes the Ackermann function and displays the intermediate values. For an extra challenge, ensure that it only displays each sub-value only once (per command call; a second `\ackermann{`m`}{`n`}` should typeset the same thing as the first one).

For example, `\ackermann{2}{2}` should display

$$\begin{aligned} A(2,2) &= A(1, A(2,1)) \\ &= A(1, A(1, A(2,0))) \\ &= A(1, A(1, A(1,1))) \\ &= A(1, A(1, A(0, A(1,0)))) \\ &= A(1, A(1, A(0, A(0,1)))) \\ &= A(1, A(1, A(0,2))) \\ &= A(1, A(1,3)) \\ &= A(1, A(0, A(1,2))) \\ &= A(1, A(0, A(0, A(1,1)))) \\ &= A(1, A(0, A(0,3))) \\ &= A(1, A(0,4)) \\ &= A(1,5) \\ &= A(0, A(1,4)) \\ &= A(0, A(0, A(1,3))) \\ &= A(0, A(0,5)) \\ &= A(0,6) \\ &= 7 \end{aligned}$$

6 Solutions

Below is the code of this LaTeX file, which will give you solutions to the simple exercises; if you copy and paste it into a new `.tex` file, it will typeset this document[1]. If you want solutions to the harder exercises, email us at M2852-teachers@esp.mit.edu.

```
\documentclass{article}
\usepackage{amsmath,amsthm,amssymb,url,verbatim,fancyvrb}
\usepackage[pdftex]{hyperref}

\usepackage{esint, mathtools}

\title{Exercises and Solutionso --
  Become a \texorpdfstring{\LaTeX}{LaTeX}er!}
\author{Jason Gross and Michele Pratusevich}
\date{Splash 2009}

\begin{document}
\VerbatimFootnotes
\maketitle
If you have trouble figuring out how to typeset one of the
following, try to figure it out using the Internet before
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stuck or frustrated, feel free to ask for help.

Note that \url{http://detexify.kirelabs.org/classify.html} is
useful.

\section{Easy}
\begin{equation} e^{i\pi} + 1 = 0 \end{equation}

\begin{equation} e^{i\theta} = \cos\theta + i\sin\theta \end{equation}

\begin{equation}
  G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}
\end{equation}

\begin{equation} x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{equation}

\begin{equation} \vec L = \vec r \times \vec p \end{equation}
```

[1] Actually, it will *almost* typeset this document. The `\input{Hard_Exercises}` line is what typesets the harder exercises, for which this file does not give you the solutions. You need to have a file `Hard_Exercises.tex` in the same folder as this .tex file if you want it to compile without any errors.

```
\begin{equation} \sqrt[3]{2} \end{equation}

\begin{equation}
  (x+y)^n = \sum_{r = 0}^n \binom{n}{r} x^r y^{n-r}
\end{equation}

\begin{equation}
 \sqrt{\frac{a_1^2 + \cdots + a_n^2}{n}} \ge % \ge is the same as \geq
  \frac{a_1 + \cdots + a_n}{n} \ge
  \sqrt[n]{a_1 \cdots a_n} \ge
  \frac{n}{\frac{1}{a_1} + \cdots + \frac{1}{a_n}}
\end{equation}

\begin{equation}
  |\langle x, y\rangle|^2 \le \langle x, x\rangle \cdot \langle y, y\rangle
\end{equation}

\begin{align}
 \text{A1: } & \varphi \longrightarrow
     (\psi \rightarrow \varphi) \nonumber \\
 \text{A2: } & (\varphi \rightarrow (\psi \rightarrow \theta))
     \longrightarrow ((\varphi \rightarrow \psi) \rightarrow
       (\varphi \rightarrow \theta)) \nonumber \\
 \text{A3: } & (\neg\varphi \rightarrow \neg\psi)
     \longrightarrow (\psi \rightarrow \varphi)
\end{align}

\newpage
\section{Medium}

\begin{equation}
 1_A = \begin{cases}
   1 & \text{if }x\in A \\
   0 & \text{if }x\notin A
  \end{cases}
\end{equation}

\begin{equation}
 n\underbrace{\uparrow\cdots\uparrow}_n n = n \rightarrow n \rightarrow n
\end{equation}

\begin{align}
  1\uparrow 1 = ^{1}1 & = 1 \nonumber \\
  2\uparrow\uparrow 2 = ^{2}2 & = 4 \nonumber \\
  3\uparrow\uparrow\uparrow 3 = ^{^{^3}3}3 & =
```

```
    3\uparrow\uparrow3\uparrow\uparrow3 =
    \underbrace{3^{3^{3^{3^{3^{3^{.^{.^{.^{3}}}}}}}}}}_{3^{3^3}\text{ threes}}
\end{align}

\begin{equation}
 \frac{{\rm d}}{{\rm d}x} f(x) =
   \lim_{\Delta x\to 0}\frac{f(x + \Delta x) - f(x)}{\Delta x}
\end{equation}

\begin{equation}
 {\rm H_2O}(\ell) + {\rm H_2O}(\ell) \rightleftharpoons
   {\rm H_3O^+}(aq) + {\rm OH^-}(aq)
\end{equation}

\begin{equation}
 \Gamma(n + 1) \stackrel{\text{\tiny def}}{=} \int_0^\infty e^{-t} t^n {\rm d}t
\end{equation}

\begin{equation}
 \gcd(n,m\bmod n);\quad x\equiv y\pmod b;
   \quad x\equiv y\mod c;\quad x\equiv y\pod d
\end{equation} \\

{In the following, note the bold symbols.
\begin{align}
 \nabla \cdot \mathbf{E} & =
   \frac{\rho}{\varepsilon_0} \nonumber \\
 \nabla \cdot \mathbf{B} & = 0 \nonumber \\
 \nabla \times \mathbf{E} & =
   -\frac{\partial \mathbf{B}}{\partial t} \nonumber \\
 \nabla \times \mathbf{B} & =
   \mu_0\mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{align}}

{For the following exercise, you will need to \verb|\usepackage{esint}|
to get the symbol $\oiint$.
\begin{align}
 \oiint_{\partial V}\mathbf{E}\cdot{\rm d}\mathbf{A} & =
   \frac{Q(V)}{\varepsilon_0} \nonumber \\
 \oiint_{\partial V}\mathbf{B}\cdot{\rm d}\mathbf{A} & = 0 \nonumber \\
 \oint_{\partial S}\mathbf{E}\cdot{\rm d}\mathbf{l} & =
   -\frac{\partial \Phi_{B, S}}{\partial t} \nonumber \\
 \oint_{\partial S}\mathbf{B}\cdot{\rm d}\mathbf{l} & =
   \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_{E, S}}{\partial t}
\end{align}}
```

```
{You might find the environments \verb|bmatrix| and \verb|pmatrix|
useful for the following exercises.

\begin{equation}
 \rho_\theta =
  \begin{pmatrix}
    \cos\theta & \sin\theta \\
    -\sin\theta & \cos\theta
  \end{pmatrix} =
  \begin{bmatrix}
    \cos\theta & \sin\theta \\
    -\sin\theta & \cos\theta
  \end{bmatrix}
\end{equation}

\begin{equation}
\left[ \begin{array}{c|ccc}
        1 & 0 & \cdots & 0 \\
 \hline 0 & * & \cdots & * \\
        \vdots & \vdots & \ddots & \vdots \\
        0 & * & \cdots & *
\end{array} \right] = \begin{array}{|c|ccc|}
 \hline 1 & 0 & \cdots & 0 \\
 \hline 0 & * & \cdots & * \\
        \vdots & \vdots &\ddots & \vdots \\
        0 & * & \cdots & * \\ \hline
\end{array}
\end{equation}}

{Note the locations of the bounds on the summation in the
following exercise.

\begin{equation}
 \sigma = \sqrt{\frac{1}{N} \sum_{i = 1}^{N} p_i(x_i - \bar x)^2} =
   \sqrt{\frac{\displaystyle \sum_{i = 1}^{N} p_i(x_i - \bar x)^2}{N}}
\end{equation}} % Use \overline{xyz} if you want the bar to span multiple characters.

\begin{equation}
 \varphi(n) = n\cdot
   \prod_{p\mid n \atop p\text{ prime}}\left(1 - \frac{1}{p}\right)
\end{equation} % use \genfrac or \substack to get rid of the warning

{If you \verb|\usepackage{mathtools}|, you can make it look like
\begin{equation}
 \varphi(n) = n\cdot \prod_{\mathclap{\substack{p\mid n \\ p\text{ prime}}}}
   \left(1 - \frac{1}{p}\right)
```

```
\end{equation}}

{Parts of the following exercise require you to \verb|\usepackage{mathtools}|.
\begin{equation}
  {}^{4}_{12}{\rm C}^{5+}_{2} \quad
  \prescript{14}{2}{{\rm C}}^{5+}_{2} \quad
  \prescript{4}{12}{{\rm C}}^{5+}_{2} \quad
  \prescript{14}{}{{\rm C}}^{5+}_{2} \quad
  \prescript{}{2}{{\rm C}}^{5+}_{2}
 \end{equation}
}

{In the following, note the size of the $/$, and the spacing
on the sides of the $\mid$.
\begin{equation*}
 \mathbb{Q} \cong \left\{\frac{a}{b}\ \middle|\ a,b\in\mathbb{Z}
   \text{ and }b\neq 0\right\} \Big/ \sim
\end{equation*}
\begin{equation} \frac{a}{b}\sim \frac{c}{d} \iff ad - bc = 0 \end{equation}}

{Notice both the horizontal and vertical spacing in the following exercise.
\begin{align}
  {1\uparrow 1} = {^{1}\!\,1} & = 1 \nonumber \\
  2\uparrow\uparrow 2 = {^{2}\!\,2} & = 4 \nonumber \\
  3\uparrow\uparrow\uparrow 3 = \text{\raisebox{0pt}[0pt]{$^{^{^3}\!3}3$}} & =
   \text{\raisebox{0pt}[0pt]{$3\uparrow\uparrow3\uparrow\uparrow3 =
   \underbrace{3^{3^{3^{3^{3^{3^{.^{.^{.^{3}}}}}}}}}}_{3^{3^3}\text{ threes}}$}}
\end{align}}

\input{Hard_Exercises}
\newpage
\section{Solutions}
Below is the code of this \LaTeX{} file, which will give you
solutions to the simple exercises; if you copy and paste it
into a new \verb|.tex| file, it will typeset this document%
\footnote{Actually, it will \emph{almost} typeset this document.
The \verb|\input{Hard_Exercises}| line is what typesets the
harder exercises, for which this file does not give you the
solutions.  You need to have a file \verb|Hard_Exercises.tex|
in the same folder as this .tex file if you want it to compile
without any errors.}.  If you want solutions to the harder
exercises, email us at \href{mailto:M2852-teachers@esp.mit.edu}
{M2852-teachers@esp.mit.edu}.

\verbatiminput{\jobname.tex}
\end{document}
```