Sumer HSSP 2013 - Relativity

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Problem Set 1

Newton, Maxwell, and Great Unifications in Physics

Problem 1 - How inertial are we?

One often considers the surface of the Earth as a good inertial frame. This is a good approximation for a lot of practical purposes, but how accurate is it?

a) Given that the Earth's radius is $6400km$, calculate the acceleration of point on the surface at latitude θ with respect to the center of the earth.

b) Given that he Earth-Sun distance is $1.5 \times 10^{11}m$ calculate the acceleration of the earth with respect to the sun.

c) Given that the sun is $26,000$ light-years away from the center of the galaxy, and that it's revolution period is 226 million years, calculate the acceleration of the sun with respect to the center of the galaxy. Which of these three effects is dominant? (A light-year is defined as the distance light moves in a year. You may use $c = 3 \times 10^8 m/s$)

Problem 2 - Gravity as a universal force

Since Ancient times, there have been measuramentes of the radius of the Earth, as well as distances from the Earth to the moon and the Earth to the sun. For an account on how these quantities were measured, see (for instance) http://galileoandeinstein.physics.virginia.edu/lectures/gkastr1.html

a) The acceleration of gravity on the surface, g, can be measured in free falling experiments, for instance. Given $g = 9.8m/s^2$, calculate the product $GM_\oplus$, where G is Newton's constant and $M_\oplus$ is the mass of the earth. (You may use that the radius of the Earth is $R_\oplus = 6400km$.)

b) Only many years after Newton proposed his law of gravity, was his constant G measured. In 1797-1798, Cavendish used a torsion balance to measure G. Cavendish attached metal spheres to the balance and measure the deflection angle to calulate the force between them. Given that the two spheres spheres were were 8.3 inches apart, their masses were 0.73 kg and 158 kg, and that he measured the gravitational force between them to be $1.74 \times 10^{-7} N$, calculate G. Use this and the answer to the previous part to calculate the mass of the Earth.

c) The previous two examples were of “terrestrial mechanics”. Let’s now apply gravity to “celestial mechanics.” The moon takes about 28 days to complete a full cycle around the Earth. Use this and previous answers to calculate the Earth-Moon distance.

d) Given that the distance between the Earth and the Sun is $1.5 \times 10^{11} m$, calculate the mass of the sun.

Problem 3 - Fun with Mawell

a) Consider a point charge at rest. Choose a suitable surface to apply **Gauss Law** and show that it implies Coulomb’s Law.

b) Since Maxwell’s equations are a set of linear differential equations, the superposition principle can be applied. The superposition principle states the electric field generated by a given charge distribution is the sum of the fields generated by its constituents. Therefore, to show that **Coulomb’s Law** implies Gauss Law, it is enough to do so for a single point charge. Show that Coulomb’s Law implies Gauss Law by considering a single point charge at rest, and using Coulomb’s law to calculate the electric flux on an arbitrary closed surface.

c) The analogous of Coulomb’s for magnetism is the Biot-Savart law, which states the the magnetic field of a current element is given by:

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \hat{r}}{r^2}, \tag{1}$$

where $\vec{dl}$ is the tangent vector to the current element with magnitude equal to the length of the current element, I is the current intensity, and $\vec{r}$ is the relative position vector (from the source to the point we are interested in). As usual $r = |\vec{r}|$ and $\hat{r} = \frac{\vec{r}}{r}$.

Use **Ampere’s Law** to prove the Biot-Savart Law for the case of an infinite straight wire.

d) Electric Induction is used to generate electric energy. Consider a square loop of side L and resistance R rotating in a uniform magnetic field $\vec{B}$. If at time $t = 0$

the field lines were normal to the loop surface, use **Faraday's Law** to calculate the current I on the loop as a function of time. Given that the electric power generated by this system is RI^2, calculate the total energy produced by time t.

e) Use Maxwell's equations to prove conservation of electric charge.

Problem 4 - Mach's Principle

Albert Einstein (1879-1955) was heavily influenced by the ideas of the Austrian physicist Ernst Mach (1838-1916). Mach believed in a stronger version of the Relativity Principle called Mach's Principle. It stated that the laws of physics should be the same not only on all inertial frames but on all frames. A famous illustration of Mach's principle is the rotating bucket of water. If we rotate a bucket of water, the surface of the water stops being flat and takes a different shape. Mach would argue that were the bucket at rest, and the universe (or the fixed stars) rotating instead, this would create a force on the bucket that would cause the surface of the water to change its shape in the same way. Calculate what shape the surface of the water on a rotating bucket would take.

Note: Even though Einstein was influenced by Mach, Mach's Principle is not true in Einstein's theories.